

# LHR-GI-12-19

Roll No \_\_\_\_\_

(To be filled in by the candidate)

(Academic Sessions 2015 – 2017 to 2017 – 2019)

**MATHEMATICS**

219-(INTER PART – II)

Time Allowed : 30 Minutes

Q.PAPER – II (Objective Type)

GROUP – 1

Maximum Marks : 20

**PAPER CODE = 8195**

Note : Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer-book. Cutting or filling two or more circles will result in zero mark in that question.

1-1	If $y = e^{2x}$ then $y_2 = :$ (A) $e^{2x}$ (B) $2e^{2x}$ (C) $4e^{2x}$ (D) $16e^{2x}$
2	$\int a^x dx = :$ (A) $\frac{\ln a}{a^x} + c$ (B) $\frac{a^x}{\ln a} + c$ (C) $\frac{1}{a^x \ln a} + c$ (D) $a^x \ln a + c$
3	$f(x) = ax + b, a \neq 0$ is : (A) Trigonometric function (B) Linear function (C) Cubic function (D) Quadratic function
4	$\int_0^{\frac{\pi}{2}} \cos x dx = :$ (A) $\frac{\pi}{2}$ (B) 0 (C) -1 (D) 1
5	$\lim_{n \rightarrow +\infty} \left(1 + \frac{1}{n}\right)^n = :$ (A) $e^{-1}$ (B) $e^2$ (C) $e^{\frac{1}{2}}$ (D) $e^3$
6	Differential of y is denoted by : (A) $\frac{dy}{dx}$ (B) dy (C) dx (D) dy'
7	If $f(x) = \cos x$ then $f'(\pi) = :$ (A) 1 (B) 0 (C) -1 (D) 2
8	The value of $\frac{dy}{dx} = \frac{-2}{x^3}$ at $x = -1$ is : (A) 4 (B) 5 (C) -2 (D) 2
9	Order of the differential equation $\frac{x d^2 y}{dx^2} + \frac{dy}{dx} - 2x = 0$ is : (A) 1 (B) 2 (C) 3 (D) 4

( Turn Over )

## SECTION – I

## 2. Write short answers to any EIGHT (8) questions :

16

- (i) Define explicit function.
- (ii) Determine whether the function  $f(x) = x\sqrt{x^2 + 5}$  is even or odd.
- (iii) Prove that  $\lim_{x \rightarrow 0} \frac{\sqrt{x+a} - \sqrt{a}}{x} = \frac{1}{2\sqrt{a}}$
- (iv) If  $y = \sqrt{x} - \frac{1}{\sqrt{x}}$ , find  $\frac{dy}{dx}$
- (v) Find  $\frac{dy}{dx}$  if  $x^2 + y^2 = 4$
- (vi) Prove that  $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
- (vii) Differentiate  $\sin^{-1} \sqrt{1-x^2}$  w.r.t. 'x'
- (viii) Differentiate  $y = a^{\sqrt{x}}$
- (ix) Prove that  $\frac{d}{dx}(\cosh x) = \sinh x$
- (x) Find  $\frac{dy}{dx}$  if  $y = (x+1)^x$
- (xi) Define decreasing function. Give an example.
- (xii) Determine  $f(x) = \cos x$  is increasing or decreasing in the interval  $(\frac{\pi}{2}, \pi)$

## 3. Write short answers to any EIGHT (8) questions :

16

- (i) What is differential coefficient?
- (ii) Evaluate  $\int \frac{e^{2x} + e^x}{e^x} dx$
- (iii) Integrate by substitution  $\int \frac{-2x}{\sqrt{4-x^2}} dx$
- (iv) Find the integral  $\int \frac{\cos x}{\sin x \ln(\sin x)} dx$
- (v) Evaluate integral by parts  $\int x \cdot \sin x dx$
- (vi) Find indefinite integral  $\int a^{ax} \left[ a \sec^{-1} x + \frac{1}{x\sqrt{x^2-1}} \right] dx$
- (vii) Evaluate  $\int \frac{5x+8}{(x+3)(2x-1)} dx$  using partial fraction.
- (viii) Define definite integral.
- (ix) Calculate the integral  $\int_0^{\frac{\pi}{4}} \sec x (\sec x + \tan x) dx$
- (x) If  $\int_{-2}^1 f(x) dx = 5$ ,  $\int_{-2}^1 g(x) dx = 4$ , then evaluate  $\int_{-2}^1 [3f(x) - 2g(x)] dx$

(Turn Over)

3. (xi) If a non-vertical line divides a plane into two, then write the name that two planes?

(xii) Graph the inequality  $x + 3y > 6$

4. Write short answers to any NINE (9) questions :

18

- (i) Find coordinates of the point that divide the join of A (-6, 3) and B (5, -2) in the ratio 2 : 3 internally.
- (ii) Show that the triangle with vertices A (1, 1), B (4, 5) and C (12, -5) is right triangle.
- (iii) Find an equation of the line through (-4, -6) and perpendicular to the line having slope  $\frac{-3}{2}$ .
- (iv) Define trapezium.
- (v) Define parabola.
- (vi) Check the position of the point (5, 6) with respect to the circle  $2x^2 + 2y^2 + 12x - 8y + 1 = 0$
- (vii) Find eccentricity of the ellipse  $x^2 + 4y^2 = 16$
- (viii) Find an equation of hyperbola if its foci (0, ±9) and directrices  $y = \pm 4$
- (ix) If  $\vec{AB} = \vec{CD}$ , find coordinates of point A. If B, C, D are (1, 2), (-2, 5), (4, 11)
- (x) Write direction cosine of  $\vec{PQ}$ , if P (2, 1, 5) Q (1, 3, 1).
- (xi) Show that vectors  $3\hat{i} - 2\hat{j} + \hat{k}$ ,  $\hat{i} - 3\hat{j} + 5\hat{k}$  and  $2\hat{i} + \hat{j} - 4\hat{k}$  form a right triangle.
- (xii) Find unit vector perpendicular to the plane of  $\underline{a}$  and  $\underline{b}$  if  $\underline{a} = -\hat{i} - \hat{j} - \hat{k}$ ,  
 $\underline{b} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ .
- (xiii) A force  $\underline{F} = 7\hat{i} + 4\hat{j} - 3\hat{k}$  is applied at P (1, -2, 3). Find its moment about the point Q (2, 1, 1)

## SECTION - II

Note : Attempt any THREE questions.

5. (a) Find the values of 'm' and 'n' so that  $f(x) = \begin{cases} mx & \text{if } x < 3 \\ n & \text{if } x = 3 \\ -2x + 9 & \text{if } x > 3 \end{cases}$

is continuous at  $x = 3$

(b) If  $y = e^x \cdot \sin x$ , then prove that  $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

6. (a) Evaluate  $\int \frac{\sqrt{2}}{\sin x + \cos x} dx$

(b) Find an equation of the perpendicular bisector of the segment joining the points A (3, 5) and B (9, 8)

7. (a) Solve the differential equation  $(x^2 - yx^2) \frac{dy}{dx} + y^2 + xy^2 = 0$

(b) Graph the solution region of the following system of linear inequalities and find the corner points :  $x + y \leq 5$ ,  $-2x + y \leq 2$ ,  $y \geq 0$

8. (a) Find the lines represented by each of the following and also find measure of the angle between them  $x^2 + 2xy \sec \alpha + y^2 = 0$

(b) Find the coordinates of the points of intersection of the line  $2x + y + 5 = 0$  and the circle  $x^2 + y^2 + 2x - 9 = 0$ . Also find the length of intercepted chord.

9. (a) Find equation of parabola with elements directrix :  $x = -2$ , focus (2, 2)

(b) Prove that  $\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$  by method of vectors.



LHR-02-12-19

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**MATHEMATICS**

219-(INTER PART – II)

Time Allowed : 30 Minutes

Q.PAPER – II ( Objective Type )

GROUP – II

Maximum Marks : 20

PAPER CODE = 8198

Note : Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer-book. Cutting or filling two or more circles will result in zero mark in that question.

1-1	$\frac{d}{dx}(\sqrt{x}) = :$ (A) $\sqrt{x}$ (B) $\frac{1}{\sqrt{x}}$ (C) $\frac{1}{2x}$ (D) $\frac{1}{2\sqrt{x}}$
2	$\int \tan x dx = :$ (A) $\ln \sec x  + c$ (B) $\ln \operatorname{cosec} x  + c$ (C) $\ln \sin x  + c$ (D) $\ln \cot x  + c$
3	$\int \frac{e^x}{e^x+3} dx = :$ (A) $\ln(e^x+3) + c$ (B) $e^{2x} + c$ (C) $e^0 + c$ (D) $e^{2x} + 3 + c$
4	$\frac{d}{dx}(\cos x^2) = :$ (A) $2x \sin x^2$ (B) $-2x \sin x^2$ (C) $2 \cos x$ (D) $-2 \sin x$
5	If $y = \sin^{-1} \frac{x}{a}$ , then $\sin y = :$ (A) $\cos y$ (B) $\cos x$ (C) $\frac{x}{a}$ (D) $\frac{y}{a}$
6	The function $y = 27 + x^2$ is a / an : (A) Constant function              (B) Even function (C) Implicit function              (D) Explicit function
7	A function $f(x)$ has relative maximum at $x = c$ , if $f'(c) = 0$ and : (A) $f''(c) > 0$ (B) $f''(c) < 0$ (C) $f''(c) = 0$ (D) $f'(c) \neq 0$
8	$\int \sec^2 x dx = :$ (A) $\cot x + c$ (B) $\tan x + c$ (C) $2 \sec x + c$ (D) $\frac{1}{\cos^2 x} + c$
9	$\int_{-\pi}^{\pi} \sin x dx = :$ (A) $2\pi$ (B) 0                      (C) 1                      (D) $\cos \pi$

( Turn Over )

LHR-CR-12-1<sup>1</sup> (2)

1-10	If $f(x) = 2x + 1$ , then $f^{-1}(x) = ?$ : (A) $2x - 1$ (B) $1 - 2x$ (C) $x - \frac{1}{2}$ (D) $\frac{x - 1}{2}$
11	y-intercept of the line $2x - y - 4 = 0$ is : (A) 2      (B) -2      (C) 4      (D) -4
12	An angle in the semi circle is of measure : (A) $30^\circ$ (B) $60^\circ$ (C) $90^\circ$ (D) $180^\circ$
13	The perpendicular distance of a line $5x + 12y = 7$ from origin is : (A) $\frac{1}{13}$ (B) $\frac{13}{7}$ (C) $\frac{7}{13}$ (D) -7
14	Equation of latus-rectum of parabola $y^2 = 4ax$ is : (A) $x = -a$ (B) $y = -a$ (C) $x = a$ (D) $y = a$
15	The mid point of line segment joining A $(-8, 3)$ , B $(2, -1)$ is : (A) $(-6, 2)$ (B) $(10, 4)$ (C) $(-3, 1)$ (D) $(-16, -3)$
16	The triple scalar product of vectors, calculates the volume of : (A) Triangle      (B) Parallelogram      (C) Tetrahedron      (D) Parallelepiped
17	The equation of line $\frac{x}{b} + \frac{y}{a} = 1$ is in : (A) Normal form      (B) Intercept form (C) Point-slope form      (D) Two-points form
18	The radius of circle $x^2 + y^2 = 5$ is : (A) 25      (B) $\sqrt{5}$ (C) 5      (D) $(0, 0)$
19	Non-zero vector $\underline{a}$ and $\underline{b}$ are parallel if $\underline{a} \times \underline{b} =$ : (A) 0      (B) 1      (C) -1      (D) $(a, b)$
20	The solution of the inequality $x + 2y < 6$ is : (A) $(1, 1)$ (B) $(1, 3)$ (C) $(1, 4)$ (D) $(1, 5)$

## 4. Write short answers to any NINE (9) questions :

- (i) Find the coordinates of the point that divides the join of A (-6, 3) and B (5, -2) internally in ratio 2 : 3.
- (ii) Find the slope and inclination of the line joining the points A (-2, 4) and B (5, 11)
- (iii) By means of slopes show that points A (-1, -3), B (1, 5) and C (2, 9) are collinear.
- (iv) Find equation of the line through (-4, 7) and parallel to the line  $2x - 7y + 4 = 0$
- (v) Find equation of circle with centre at (5, -2) and radius 4.
- (vi) Find focus and vertex of the parabola  $y^2 = -8(x-3)$
- (vii) Find equation of tangent to the parabola  $x^2 = 16y$  at the point whose abscissa is 8.
- (viii) Find foci and vertices of the ellipse  $25x^2 + 9y^2 = 225$
- (ix) Find the angle between the vectors  $\underline{u} = 2\hat{i} - \hat{j} + \hat{k}$  and  $\underline{v} = -\hat{i} + \hat{j}$
- (x) Find scalar  $\alpha$  so that the vectors  $2\hat{i} + \alpha\hat{j} + 5\hat{k}$  and  $3\hat{i} + \hat{j} + \alpha\hat{k}$  are perpendicular.
- (xi) If  $\underline{v}$  is a vector for which  $\underline{v} \cdot \hat{i} = 0$ ,  $\underline{v} \cdot \hat{j} = 0$ ,  $\underline{v} \cdot \hat{k} = 0$  find  $\underline{v}$
- (xii) Prove that  $\underline{a} \times (\underline{b} + \underline{c}) + \underline{b} \times (\underline{c} + \underline{a}) + \underline{c} \times (\underline{a} + \underline{b}) = 0$
- (xiii) Find the value of  $\alpha$  so that  $\alpha\hat{i} + \hat{j}$ ,  $\hat{i} + \hat{j} + 3\hat{k}$  and  $2\hat{i} + \hat{j} - 2\hat{k}$  are coplanar.

## SECTION - II

Note : Attempt any THREE questions.

5. (a) If  $f(x) = \begin{cases} 3x & \text{if } x \leq -2 \\ x^2 - 1 & \text{if } -2 < x < 2 \\ 3 & \text{if } x \geq 2 \end{cases}$

discuss continuity at  $x = 2$  and  $x = -2$ 

(b) If  $y = e^x \sin x$ , show that  $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

6. (a) Integrate  $\int \frac{12}{x^3 + 8} dx$

(b) Find equations of two parallel lines, perpendicular to  $2x - y + 3 = 0$  such that the product of the x- and y-intercepts of each is 3.

7. (a) Evaluate the definite integral  $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x(2 + \sin x)} dx$

(b) Minimize  $z = 2x + y$  subject to the constraints  $x + y \geq 3$ ,  $7x + 5y \leq 35$ ,  $x \geq 0$ ,  $y \geq 0$

8. (a) Find equation of the line through the point (2, -9) and intersection of the lines  $2x + 5y - 8 = 0$  and  $3x - 4y - 6 = 0$

(b) Show that the circles  $x^2 + y^2 + 2x - 2y - 7 = 0$  and  $x^2 + y^2 - 6x + 4y + 9 = 0$  touch externally.

9. (a) Find an equation of the ellipse having foci  $(\pm 5, 0)$  and passing through the point  $(\frac{2}{3}, \sqrt{3})$

(b) A particle acted upon by constant forces  $4\hat{i} + \hat{j} - 3\hat{k}$  and  $3\hat{i} - \hat{j} - \hat{k}$  is displaced from A (1, 2, 3) to B (5, 4, 1). Find the work done.