Roll No _____ (To be filled in by the candidate)

(Academic Sessions 2019 – 2021 to 2021 – 2023)

MATHEMATICS 223-1st Annual-(INTER PART – II) Time Allowed : 30 Minutes

Q.PAPER – II (Objective Type) GROUP – I Maximum Marks : 20

PAPER CODE = 8191 LHR-12-1-23

Note : Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer-book. Cutting or filling two or more circles will result in zero mark in that question.

1-1	The perimeter P of a square as a function of its area A is given as :			
	(A) $4A$	(B) $4\sqrt{A}$	(C) $2A$	(D) $2\sqrt{A}$
2	Domain of cosine function $y = cosx$ is :			
	(A) Real numbers	(B) $[-1,1]$	(C) $(0,\infty)$	(D) $]-1,1[$
3	If $y = \tanh^{-1} x$, then $\frac{dy}{dx} =$:			
	(A) $\frac{1}{1+x^2}$	(B) $\frac{1}{1-x^2}$	(C) $\frac{-1}{1+x^2}$	(D) $\frac{-1}{1-x^2}$
4	$\frac{d}{dx}(a^{\lambda x}) =$:			
	(A) $a^{\lambda x}$	(B) $a^{\lambda x}\ell na$	(C) $\lambda a^{\lambda x}\ell na$	(D) $\frac{a^{\lambda x}}{\lambda \ell na}$
5	$\frac{d}{dx}(\sin\sqrt{x}) =$:			
	(A) $\cos\sqrt{x}$	(B) $\cos\sqrt{x} \cdot \frac{1}{\sqrt{x}}$	(C) $\sqrt{x}\cos\sqrt{x}$	(D) $\cos\sqrt{x} \cdot \frac{1}{2\sqrt{x}}$
6	If $y = x^2 - 1$, then $dy =$:			
	(A) $xdx + c$	(B) $(x-1)dx$	(C) $2x\,dx + c$	(D) $2x\,dx$
7	$\int_0^3 \frac{dx}{x^2+9} =$:			
	(A) $\frac{\pi}{4}$	(B) $\frac{-\pi}{4}$	(C) 0	(D) $\frac{\pi}{12}$
8	$\int e^x(\sin x + \cos x)\,dx =$:			
	(A) $e^x \cos x + c$	(B) $e^x \sin x$		
	(C) $e^x \sin x + c$	(D) $e^x \cos x$		
9	$\int \frac{2}{x+2}dx =$:			
	(A) $\ell n\lvert x+2\rvert + c$	(B) $\ell n\lvert x+2\rvert^2 + c$	(C) $\frac{1}{\ell n\lvert x+2\rvert} + c$	(D) $2\ell nx + c$
10	$\int \frac{1}{\cos^2 x}dx =$:			
	(A) $\frac{1}{\sin^2 x} + c$	(B) $\tan x + c$	(C) $\sec^2 x + c$	(D) $\operatorname{cosec}^2 x + c$

(Turn Over)

11	The lines represented by $ax^2+2hxy+by^2=0$ are imaginary if : (A) $h^2-ab=0$ (B) $h^2-ab<0$ (C) $h^2-ab>0$ (D) $h^2-ab\neq 0$
12	Two lines $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ are parallel if : (A) $a_1a_2+b_1b_2=0$ (B) $a_1a_2-b_1b_2=0$ (C) $a_1b_2-a_2b_1=0$ (D) $a_1b_2+a_2b_1=0$
13	Inclination of the line joining the points (4 , 6) and (4 , 8) is : (A) 90° (B) 45° (C) 30° (D) Undefined
14	A region is said to feasible region which is restricted to : (A) I quadrant (B) II quadrant (C) III quadrant (D) IV quadrant
15	An angle in a semicircle is of measure : (A) 90° (B) 60° (C) 45° (D) 30°
16	The coordinate of the vertices of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is : (A) $(0,\pm b)$ (B) $(\pm b,0)$ (C) $(0,\pm a)$ (D) $(\pm a,0)$
17	Focus of the parabola $x^2-5y=0$ is : (A) $(\frac{5}{4},0)$ (B) $(0,\frac{5}{4})$ (C) $(0,\frac{-5}{4})$ (D) $(-\frac{5}{4},0)$
18	For parabola, value of eccentricity e is : (A) $e=0$ (B) $e<1$ (C) $e>1$ (D) $e=1$
19	If $\underline{u},\underline{v}$ and $\underline{w}$ are coterminous edges of a tetrahedron, then its volume is : (A) $[\underline{u}\ \underline{v}\ \underline{w}]$ (B) $\frac{1}{3}[\underline{u}\ \underline{v}\ \underline{w}]$ (C) $\frac{1}{6}[\underline{u}\ \underline{v}\ \underline{w}]$ (D) $\frac{1}{9}[\underline{u}\ \underline{v}\ \underline{w}]$
20	A vector perpendicular to both vectors $\underline{a}$ and $\underline{b}$ is : (A) $\underline{a}\cdot\underline{b}$ (B) $\underline{a}\times\underline{b}$ (C) $\frac{\underline{a}\cdot\underline{b}}{\lvert\underline{a}\rvert}$ (D) $\underline{b}.\underline{a}$

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MATHEMATICS 223-1st Annual-(INTER PART – II) Time Allowed : 2.30 hours

PAPER – II (Essay Type) GROUP – I Maximum Marks : 80

SECTION – I L HD-12-1-23

2. Write short answers to any EIGHT (8) questions : **16**

(i) For $f(x)=\frac{2x+1}{x-1}$, find $f^{-1}(x)$

(ii) Evaluate $\lim_{\theta\to 0}\frac{1-\cos\theta}{\sin\theta}$

(iii) Discuss the continuity of $f(x)$ at $x = c = 2$, $f(x)=\begin{cases}2x+5 & \text{if } x\le 2\\ 4x+1 & \text{if } x>2\end{cases}$

(iv) Differentiate w.r.t ' x ' $(x-5)(3-x)$

(v) Find $\frac{dy}{dx}$ if $y^2-xy-x^2+4=0$

(vi) Differentiate w.r.t. ' θ ' $(\sin 2\theta-\cos 3\theta)^2$

(vii) Find $\frac{dy}{dx}$ if $y=x^2\ell n\frac{1}{x}$

(viii) Find y_4 if $y=(2x+5)^{3/2}$

(ix) Apply Maclaurin series expansion to prove that $\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+-----$

(x) Find extreme values for $f(x)=x^2-x-2$

(xi) Define feasible region.

(xii) Graph the inequality $x+2y\le 6$

3. Write short answers to any EIGHT (8) questions : **16**

(i) Find δy and dy in the case $y=x^2+2x$ when x changes from 2 to 1.8

(ii) Evaluate $\int\frac{1}{\sqrt{x}(\sqrt{x}+1)}dx$, $x>0$

(iii) Evaluate $\int a^{x^2}x\,dx$ $(a>0, a\neq 1)$

(iv) Evaluate $\int\sqrt{4-5x^2}\,dx$

(v) Evaluate $\int_0^{\frac{\pi}{6}} x\cos x\,dx$

(vi) Find area below the curve $y=3\sqrt{x}$ and above the x-axis between $x=1$ and $x=4$

(vii) Solve the differential equation $x^2(2y+1)\frac{dy}{dx}-1=0$

(viii) Find the position vector of the point of division of the line segments joining the following pair of points, in the given ratio, point C with position vector $2\underline{i}-3\underline{j}$ and point D with position vector $3\underline{i}+2\underline{j}$ in the ratio 4 : 3

(ix) If $\underline{u}=2\underline{i}+3\underline{j}+4\underline{k}$, $\underline{v}=-\underline{i}+3\underline{j}-\underline{k}$ *and* $\underline{w}=\underline{i}+6\underline{j}+z\underline{k}$ represent the sides of a triangle, find the value of z.

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5. (x) Find the angle between the vectors $\underline{u}=2\underline{i}-\underline{j}+\underline{k}$ and $\underline{v}=-\underline{i}+\underline{j}$

(xi) If $\underline{a}=4\underline{i}+3\underline{j}+\underline{k}$ and $\underline{b}=2\underline{i}-\underline{j}+2\underline{k}$, find a unit vector perpendicular to both $\underline{a}$ and $\underline{b}$. Also find the sine of angle between the vectors $\underline{a}$ and $\underline{b}$.

(xii) Find the area of the triangle with vertices A (1 , – 1 , 1) , B (2 , 1, – 1) and C (– 1, 1, 2)

4. Write short answers to any NINE (9) questions : **18**

(i) Show that the points A (0, 2) , B $(\sqrt{3}, -1)$ and C (0, – 2) are vertices of a right triangle.

(ii) Find k so that the line joining A (7 , 3) , B (k , – 6) and line joining C (– 4 , 5) , D (– 6 , 4) are parallel.

(iii) Find an equation of line if its slope is 2 and y-intercept is 5.

(iv) Transform the equation $5x - 12y + 39 = 0$ into two-intercept form.

(v) Find the distance from the points P (6 , – 1) to the line $6x - 4y + 9 = 0$

(vi) Find the point of intersection of lines $3x + y + 12 = 0$ and $x + 2y - 1 = 0$

(vii) Find the angle between the lines represented by $x^2 - xy - 6y^2 = 0$

(viii) Find an equation of circle with centre at $(\sqrt{2}, -3\sqrt{3})$ and radius $2\sqrt{2}$

(ix) Find centre and radius of circle $x^2 + y^2 + 12x - 10y = 0$

(x) Find vertex and directrix of parabola $x^2 = 16y$

(xi) Find the focus and vertex of parabola $x^2 = 4(y-1)$

(xii) Find centre and foci of $4x^2 + 9y^2 = 36$

(xiii) Find eccentricity and vertices of $\frac{y^2}{16} - \frac{x^2}{9} = 1$

SECTION – II

Note : Attempt any THREE questions.

5. (a) Evaluate $\lim\limits_{\theta \to 0} \frac{1-\cos p\theta}{1-\cos q\theta}$ 5

(b) If $\frac{y}{x} = \tan^{-1}\frac{x}{y}$ then prove that $\frac{dy}{dx} = \frac{y}{x}$ 5

6. (a) Evaluate $\int \frac{x}{x^4 + 2x^2 + 5}\, dx$ 5

(b) Find equations of two parallel lines perpendicular to $2x - y + 3 = 0$ such that the product of the x-intercept and y-intercept of each is 3. 5

7. (a) Evaluate $\int_0^{\frac{\pi}{4}} (1+\cos^2\theta)\tan^2\theta\, d\theta$ 5

(b) Minimize $z = 2x + y$ subject to the constraints
$x + y \geq 3$, $7x + 5y \leq 35$, $x \geq 0$, $y \geq 0$ 5

8. (a) If $y = (\cos^{-1}x)^2$, prove that $(1-x^2)y_2 - xy_1 - 2 = 0$ 5

(b) Find equations of the tangents to the circle $x^2 + y^2 = 2$ and parallel to the line $x - 2y + 1 = 0$ 5

9. (a) Find volume of the tetrahedron with the vertices (0, 1, 2) , (3, 2, 1) (1, 2, 1) and (5, 5, 6) 5

(b) Find the centre, foci, eccentricity and directrices of ellipse $\frac{(2x-1)^2}{4} + \frac{(y+2)^2}{16} = 1$ 5

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MATHEMATICS 223-1st Annual-(INTER PART – II) Time Allowed : 30 Minutes

Q.PAPER – II (Objective Type) GROUP – II Maximum Marks : 20

PAPER CODE = 8194 LHR-12-2-23

Note : Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with Marker or Pen ink in the answer-book. Cutting or filling two or more circles will result in zero mark in that question.

1-1	The mid point of the line segment joining the foci of an ellipse is called : (A) Vertex (B) Directrix (C) Centre (D) Minor axis
2	If (3 , 5) is mid point of (5 , a) and (b, 7) then : (A) $a=4\,,\,b=2$ (B) $a=3\,,\,b=3$ (C) $a=7\,,\,b=-2$ (D) $a=3\,,\,b=1$
3	If 2 and 2 are x and y components of a vector, then its angle with x-axis is : (A) 30° (B) 60° (C) 45° (D) 90°
4	(3 , 2) is not a solution of the inequality : (A) $x-y>1$ (B) $x+y>2$ (C) $3x+5y>8$ (D) $3x-7y<3$
5	$\underline{i}\times\underline{j}=$: (A) $\underline{k}$ (B) $\underline{i}$ (C) $-\underline{k}$ (D) $\underline{j}$
6	Slope of line $3x-2y+5=0$ is : (A) $\frac{-2}{3}$ (B) $\frac{2}{3}$ (C) $\frac{3}{2}$ (D) $\frac{-3}{2}$
7	Length of the diameter of the circle $(x+5)^2+(y-8)^2=12$: (A) $2\sqrt{3}$ (B) 12 (C) 24 (D) $4\sqrt{3}$
8	Transverse axis of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is : (A) $x=\frac{a}{e}$ (B) $y=0$ (C) $x=0$ (D) $y=\frac{a}{e}$
9	Equation of line in slope intercept form is : (A) $y=mx+c$ (B) $\frac{x}{a}+\frac{y}{b}=1$ (C) $y-y_1=m(x-x_1)$ (D) $x\cos\alpha+y\sin\alpha=p$
10	The condition for a line $y=mx+c$ to be tangent to the circle $x^2+y^2=a^2$ is that : (A) $c=\pm m\sqrt{1+a^2}$ (B) $c=\pm a\sqrt{1+m^2}$ (C) $c=\pm a\sqrt{1-m^2}$ (D) $c=\pm\sqrt{1-m^2}$

(Turn Over)

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11	$f(x)=f(o)+xf'(o)+\frac{x^2}{2!}f''(o)+\frac{x^3}{3!}f'''(o)+----$ is called : (A) Taylor's series (B) Binomial series (C) Maclaurin's series (D) Laurent series
12	$\lim_{x\to 0}\frac{\sin 7x}{x}=:$ (A) 7 (B) -7 (C) $\frac{-1}{7}$ (D) $\frac{1}{7}$
13	$\int \frac{e^x}{e^x+1}dx=:$ (A) $\ell n(e^x+1)+c$ (B) $\ell n e^x+c$ (C) $e^{-x}+c$ (D) e^x+c
14	$\frac{d}{dx}(x^2+1)^2=:$ (A) $2x(x^2+1)$ (B) $\frac{(x^2+1)^3}{3}$ (C) $2(x^2+1)$ (D) $4x(x^2+1)$
15	$\int \sin^2 x\,dx=:$ (A) $\frac{x}{2}-\frac{\sin 2x}{4}+c$ (B) $\frac{x}{2}+\frac{\sin 2x}{4}+c$ (C) $\frac{x}{2}-\frac{\sin 2x}{2}+c$ (D) $\frac{x}{2}+\frac{\sin 2x}{2}+c$
16	$\frac{d}{dx}(\tan x^2)=:$ (A) $\sec^2 x^2$ (B) $2x\sec^2 x^2$ (C) $-\sec^2 x^2$ (D) $-2x\sec x^2$
17	$\int e^x(\cos x-\sin x)\,dx=:$ (A) $e^x\sin x+c$ (B) $e^x\cos x+c$ (C) $e^x\tan x+c$ (D) $e^x\cot x+c$
18	If $f(x)=\sin x+\cos x$ then $f(x)$ is : (A) Even function (B) Odd function (C) Neither even nor odd (D) Constant function
19	$\int_0^3 \frac{1}{9+x^2}dx=:$ (A) $\frac{\pi}{4}$ (B) $\frac{-\pi}{12}$ (C) $\frac{-\pi}{4}$ (D) $\frac{\pi}{12}$
20	$\frac{d}{dx}(f(x)\sin x)=:$ (A) $f(x)\cos x+f'(x)\sin x$ (B) $f'(x)\sin x-f(x)\cos x$ (C) $f'(x)\cos x$ (D) $f'(x)\cos x+f(x)\sin x$

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MATHEMATICS 223-1st Annual-(INTER PART – II) Time Allowed : 2.30 hours

PAPER – II (Essay Type) GROUP – II Maximum Marks : 80

SECTION – I CHR-12-2-23

2. Write short answers to any EIGHT (8) questions : **16**

(i) Find the domain and range of $f(x)=\sqrt{x^2-4}$

(ii) Show that $x=a\sec\theta$, $y=b\tan\theta$ represents the equation of hyperbola.

(iii) If $f(x)=-2x+8$, find $f^{-1}(x)$ and $f^{-1}(-1)$

(iv) Differentiate $(3-x)(x-5)$ w.r.t ' x '

(v) Find derivative of $\sqrt{\frac{1+x}{1-x}}$

(vi) If $y=x^4+2x^2+2$, prove $\frac{dy}{dx}=4x\sqrt{y-1}$

(vii) Find the derivative of $(x^3+1)^9$ w.r.t. ' x '

(viii) Find $\frac{dy}{dx}$ if $y^3-2xy^2+x^2y+3x=0$

(ix) Differentiate w.r.t. variable involved of $\tan^3\theta\sec^2\theta$

(x) Find $\frac{dy}{dx}$ if $y=a^x$

(xi) Define feasible region.

(xii) Graph the feasible region $2x-3y\leq 6 \quad x\geq 0$, $y\geq 0$

3. Write short answers to any EIGHT (8) questions : **16**

(i) Using differentials to find $\frac{dy}{dx}$ if $xy-\ell nx=c$

(ii) Evaluate $\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx$

(iii) Evaluate $\int\frac{e^x}{e^x+3}dx$

(iv) Evaluate $\int \ell nx\,dx$

(v) Evaluate $\int_{-6}^{2}\sqrt{3-x}\,dx$

(vi) Find the area bounded by cos function from $x=-\frac{\pi}{2}$ to $x=\frac{\pi}{2}$

(vii) Solve the differential equation $\sec^2 x\tan y\,dx+\sec^2 y\tan x\,dy=0$

(viii) Find the magnitude of the vector $\underline{u}=\hat{i}+\hat{j}$

(ix) Find direction cosines of $\vec{v}=3\hat{i}-\hat{j}+2\hat{k}$

(x) Calculate the projection of $\vec{b}$ along $\vec{a}$ if $\vec{a}=\hat{i}-\hat{k}$; $\vec{b}=\hat{j}+\hat{k}$

(xi) If $\vec{a}=2\hat{i}+\hat{j}-\hat{k}$; $\vec{b}=\hat{i}-\hat{j}+\hat{k}$, find $\vec{b}\times\vec{a}$

(xii) Prove that the vectors $\hat{i}-2\hat{j}+3\hat{k}$, $-2\hat{i}+3\hat{j}-4\hat{k}$ and $\hat{i}-3\hat{j}+5\hat{k}$ are coplanar.

(Turn Over)

LHR-12-2-23

4. Write short answers to any NINE (9) questions : **18**

(i) Find the equation of the straight line whose slope is 2 and y-intercept is 5.

(ii) Using slopes, show that the triangle with its vertices A (6, 1) , B (2, 7) and C (– 6, – 7) is a right triangle.

(iii) Find an equation of the line through (– 4 , 7) and parallel to the line $2x - 7y + 4 = 0$

(iv) Find h such that A (– 1 , h), B (3 , 2) and C (7 , 3) are collinear.

(v) Write intercepts form of equation of straight line.

(vi) Check whether the following lines are concurrent or not

$3x - 4y - 3 = 0$

$5x + 12y + 1 = 0$

$32x + 4y - 17 = 0$

(vii) Find the slope and inclination of the line joining points (– 2 , 4) and (5, 11)

(viii) Find an equation of circle with centre at $(\sqrt{2}\ , -3\sqrt{3})$ and radius $2\sqrt{2}$

(ix) Define focus and directrix of the parabola.

(x) Find the centre and foci of the ellipse $x^2 + 4y^2 = 16$

(xi) Find equation of tangent to $y^2 = 4ax$ at (x_1, y_1)

(xii) Show that the equation $5x^2 + 5y^2 + 24x + 36y + 10 = 0$ represents a circle. Find its centre.

(xiii) Find an equation of the ellipse with given data : Foci (0 , – 1) and (0 , – 5) and major axis of length 6.

SECTION – II

Note : Attempt any THREE questions.

5. (a) If θ is measured in radians then prove that $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$ 5

(b) Find $\frac{dy}{dx}$ if $y = (1 + 2\sqrt{x})^3 \cdot x^{\frac{3}{2}}$ 5

6. (a) Evaluate $\int \ell n(x + \sqrt{x^2 + 1})\, dx$ 5

(b) Find equations of two parallel lines perpendicular to $2x - y + 3 = 0$ such that the product of the x-intercept and y-intercept of each is 3. 5

7. (a) Solve the differential equation $2e^x \tan y\, dx + (1 - e^x)\sec^2 y\, dy = 0$ 5

(b) Maximize $f(x, y) = x + 3y$ subject to constraints

$2x + 5y \leq 30$, $5x + 4y \leq 20$, $x \geq 0$, $y \geq 0$ 5

8. (a) If $y = e^x \sin x$, show that $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$ 5

(b) Find equations of tangents to the circle $x^2 + y^2 = 2$ perpendicular to the line $3x + 2y = 6$ 5

9. (a) Show that the equation $9x^2 - 18x + 4y^2 + 8y - 23 = 0$ represents an ellipse. Find its elements and sketch its graph. 5

(b) Prove that in any triangle ABC $c = a\cos B + b\cos A$ 5