

SGD-12-1-23

1223 Warning:- Please write your Roll No. in the space provided and sign. Roll No.-----
(Inter Part - II) (Session 2019-21 to 2021-23) Sig. of Student -----

Mathematics (Objective)

(Group 1st)

Paper (II)

Time Allowed:- 30 minutes

PAPER CODE 4197

Maximum Marks:- 20

Note:- You have four choices for each objective type question as A, B, C and D. The choice which you think is correct; fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling two or more circles will result in zero mark in that question. Write PAPER CODE, which is printed on this question paper, on the both sides of the Answer Sheet and fill bubbles accordingly, otherwise the student will be responsible for the situation. Use of Ink Remover or white correcting fluid is not allowed.

Q. 1

1) $\frac{d}{dx} \tan^{-1} x = \underline{\hspace{2cm}}$

(A) $\frac{1}{1+x^2}$

(B) $\frac{-1}{1+x^2}$

(C) $\frac{1}{\sqrt{1+x^2}}$

(D) $\frac{-1}{\sqrt{1+x^2}}$

2) $\int_0^{\frac{\pi}{6}} \frac{3}{x^2+9} dx$

(A) $\frac{\pi}{6}$

(B) $\frac{3\pi}{4}$

(C) $\frac{\pi}{12}$

(D) $\frac{\pi}{4}$

3) $\int \sec x \tan x dx$

(A) $\tan x + c$

(B) $\sec^2 x + c$

(C) $\sec x + c$

(D) $\tan^2 x + c$

4) $y = x^2 + 2x - 1$ is _____ function.

(A) Constant

(B) Linear

(C) Implicit

(D) Explicit

5) $f \circ f^{-1}(x)$ is _____ function.

(A) Constant

(B) Identity

(C) Even

(D) Exponential

6) Value of dy , for $y = x^2$ and x changes from 2 to 2.1

(A) 0.4

(B) 0.2

(C) 0.1

(D) 0

7) $f(x) = x^{2/3}$, Then $f'(8) = \underline{\hspace{2cm}}$

(A) $\frac{3}{2}$

(B) $\frac{2}{3}$

(C) $\frac{1}{3}$

(D) 3

8) $\frac{d}{dx} e^{3x} = \underline{\hspace{2cm}}$

(A) e^{3x}

(B) $3e^{3x}$

(C) $\frac{e^{3x}}{3}$

(D) e^x

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9) Length of transverse axis of $\frac{x^2}{9} - \frac{y^2}{4} = 1$

- (A) 3 (B) 6 (C) 2 (D) 4

10) If $\underline{u} = \underline{v}$, Then $\underline{u} \cdot \underline{v} \times \underline{w} =$ _____

- (A) 1 (B) 0 (C) -1 (D) ∞

11) Length of vector $2\underline{i} - \underline{j} - 2\underline{k}$ is

- (A) 0 (B) 2 (C) 3 (D) 4

12) $\int e^x (\ln x + \frac{1}{x}) dx$

- (A) $\frac{e^x}{x} + c$ (B) $e^x + c$ (C) $e^x \ln x + c$ (D) $\ln x + c$

13) $\int \tan \frac{\pi}{4} dx$

- (A) $\ln \sin \frac{\pi}{4} + c$ (B) $\sec^2 \frac{\pi}{4} + c$ (C) $\frac{x}{4} + c$ (D) $x + c$

14) Mid point of A(1,2) and B (5,4) is

- (A) (3,3) (B) (2,1) (C) (3,2) (D) (2,3)

15) Slope of line joining A(3,1) and B (4,7) is

- (A) $\frac{6}{7}$ (B) 6 (C) $\frac{4}{3}$ (D) $\frac{7}{3}$

16) Equation of horizontal line through (3,4)

- (A) $y = 3$ (B) $y = 4$ (C) $x = 3$ (D) $x = 4$

17) (1,0) is solution of _____

- (A) $2x + 3y \geq 3$ (B) $2x - 3y \geq 3$ (C) $2x + y \geq 1$ (D) $x - 3y \geq 2$

18) Equation of latus rectum of $y^2 = 4x$ is

- (A) $y = -2$ (B) $y = 2$ (C) $x = -2$ (D) $x = 2$

19) Radius of circle $x^2 + y^2 + 2y = 5$ is

- (A) $\sqrt{6}$ (B) $\sqrt{5}$ (C) 4 (D) 2

20) Foci of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

- (A) $(\pm c, 0)$ (B) $(0, \pm a)$ (C) $(\pm a, 0)$ (D) $(0, \pm b)$

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Section ----- I

2. Answer briefly any Eight parts from the followings:- 8 × 2 = 16

- (i) Prove the identity $\cosh^2 x + \sinh^2 x = \cosh 2x$
- (ii) Prove that $\lim_{x \rightarrow 0} \frac{\sqrt{x+a} - \sqrt{a}}{x} = \frac{1}{2\sqrt{a}}$
- (iii) Evaluate $\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta}$ (iv) If $y = x^4 + 2x^2 + 2$ Prove that $\frac{dy}{dx} = 4x\sqrt{y-1}$
- (v) Differentiate $x^2 + \frac{1}{x^2}$ w.r.t $x - \frac{1}{x}$
- (vi) Prove that $\frac{d}{dx}(a^x) = a^x \ln a$ by ab-initio method. (vii) Differentiate $(\ln x)^x$ w.r.t. x
- (viii) If $y = \sin^{-1} \frac{x}{a}$, then show that $y_2 = x(a^2 - x^2)^{-3/2}$ (ix) Expand $(1+x)^n$ in the Maclaurin Series
- (x) Determine the intervals in which f is increasing or decreasing if $f(x) = x^3 - 6x^2 + 9x$.
- (xi) Define convex region and corner point.
- (xii) Graph the solution region of the following system of linear inequalities and find the corner points.
 $2x - 3y \leq 6$
 $2x + 3y \leq 12$
 $x \geq 0$

3. Answer briefly any Eight parts from the followings:- 8 × 2 = 16

- (i) If $y = \sqrt{x}$ find δy when x changes from 4 to 4.41. (ii) Evaluate $\int \frac{\sqrt{y(y+1)}}{y} dy$
- (iii) Evaluate $\int \frac{2x}{\sqrt{4-x^2}} dx$ (iv) Evaluate $\int \tan^{-1} x dx$ (v) Evaluate $\int \frac{x e^x}{(1+x)^2} dx$
- (vi) Evaluate $\int_2^{\sqrt{5}} x\sqrt{x^2-1} dx$ (vii) Find the area bounded by Cos function from $x = \frac{\pi}{2}$ to $x = \frac{-\pi}{2}$.
- (viii) Find magnitude and direction cosines of $\underline{v} = 2\hat{i} + 3\hat{j} - 4\hat{k}$.
- (ix) Calculate the projection of $\underline{a} = [3, 1, -1]$ along $\underline{b} = [-2, -1, 1]$.
- (x) If $\underline{a} + \underline{b} + \underline{c} = 0$ then prove that $\underline{b} \times \underline{c} = \underline{c} \times \underline{a}$. (xi) Find the value of $2\hat{i} \times 2\hat{j} \cdot \hat{k}$.
- (xii) Prove that $\underline{u} \cdot (\underline{v} \times \underline{w}) + \underline{v} \cdot (\underline{w} \times \underline{u}) + \underline{w} \cdot (\underline{u} \times \underline{v}) = 3\underline{u} \cdot (\underline{v} \times \underline{w})$

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4. Answer briefly any Nine parts from the followings:- $9 \times 2 = 18$
- (i) Show that the points A(-1,2), B(7,5) and C(2,-6) are vertices of a right angle triangle.
 - (ii) Check whether the origin and point (5,-8) lies on same or opposite side of the line $3x + 7y + 15 = 0$
 - (iii) Find area of the region bounded by the triangle with vertices $(a, b+c)$, $(a, b-c)$ and $(-a, c)$.
 - (iv) Find k so that the line joining A(7, 3), B(k,-6) and the line joining C(-4,5), D(-6,4) are parallel.
 - (v) Find equation of line passing through (-8, 5) and having slope undefined.
 - (vi) Find measure of angle between the lines represented by $6x^2 - 19xy + 15y^2 = 0$
 - (vii) Find the distance of the point P(6,-1) to the line $6x - 4y + 9 = 0$.
 - (viii) Find equation of circle with ends of a diameter at (-3,2) and (5,-6).
 - (ix) Write equation of tangent to the circle $x^2 + y^2 = 25$ at (4,3).
 - (x) Find centre and vertex of the Parabola $y^2 = -8(x-3)$.
 - (xi) Find centre and foci of the ellipse $9x^2 + y^2 = 18$
 - (xii) Find an equation of ellipse with given foci $(-3\sqrt{3}, 0)$ and vertices $(\pm 6, 0)$.
 - (xiii) Find eccentricity and coordinates of the vertices of the hyperbola $\frac{y^2}{16} - \frac{x^2}{49} = 1$

Section ----- II

Note: Attempt any three questions.

(10 × 3 = 30)

- 5 -(a) Evaluate $\lim_{\theta \rightarrow 0} \frac{\tan \theta - \sin \theta}{\sin^3 \theta}$
- (b) Differentiate $\frac{x^2 + 1}{x^2 - 1}$ w.r.t. $\frac{x-1}{x+1}$
- 6 -(a) Evaluate $\int \sqrt{x^2 - a^2} dx$
- (b) Find an equation of the perpendicular bisector of the segment joining the points A(3,5) and B (9,8).
- 7 -(a) Find the area between the x -axis and the curve $y = \sqrt{2ax - x^2}$, when $a > 0$
- (b) Maximize $f(x, y) = x + 3y$ subject to constraints $2x + 5y \leq 30$, $5x + 4y \leq 20$, $x \geq 0$, $y \geq 0$
- 8 -(a) Show that $y = \frac{\ln x}{x}$ has maximum value at $x = e$.
- (b) Find an equation of the circle passing through A(3,-1), B(0,1) and having centre at $4x - 3y - 3 = 0$.
- 9 -(a) Find the focus, vertex and directrix of the parabola $x^2 - 4x - 8y + 4 = 0$.
- (b) By using vectors, prove that $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$.

Mathematics (Objective)**(Group II) SGD-12-2-23 Paper (II)**

Time Allowed:- 30 minutes

PAPER CODE 4196

Maximum Marks:- 20

Note:- You have four choices for each objective type question as A, B, C and D. The choice which you think is correct; fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling two or more circles will result in zero mark in that question. Write **PAPER CODE**, which is printed on this question paper, on the both sides of the Answer Sheet and fill bubbles accordingly, otherwise the student will be responsible for the situation. Use of Ink Remover or white correcting fluid is not allowed.

Q. 1

1) $\int \frac{1}{x^2} dx =$

(A) $\ln x + c$

(B) $\ln x^2 + c$

(C) $\frac{-2}{x^3} + c$

(D) $\frac{-1}{x} + c$

2) $\int_0^{\frac{3\pi}{2}} \cos x dx =$

(A) 0

(B) 1

(C) -1

(D) 2

3) $x = a \cos \theta$, $y = b \sin \theta$ are parametric equations of

(A) Circle

(B) Parabola

(C) Ellipse

(D) Hyperbola

4) If $f(x) = \sqrt{x^2 - 1}$ then Domain of f is

(A) $(-\infty, \infty)$

(B) $[1, \infty)$

(C) $[0, \infty)$

(D) $(-\infty, -1] \cup [1, \infty)$

5) If $y = \frac{1}{x^2}$ then $\frac{dy}{dx}$ at $x = -1$ is

(A) 2

(B) 3

(C) $\frac{1}{3}$

(D) 4

6) $(1+x^2) \frac{d}{dx} (\tan^{-1} x + \cot^{-1} x) =$

(A) 2

(B) $\frac{2}{1+x^2}$

(C) 0

(D) $\frac{-2}{1+x^2}$

7) If $f(x+h) = a^{x+h}$ then $f'(x) =$

(A) $a^{x+h} \ln(x+h)$

(B) $a^x \ln a$

(C) $a^x \ln x$

(D) $a^{x+h} \ln a$

8) $\frac{d}{dx} (\sinh^{-1} x) =$

(A) $\frac{1}{\sqrt{1+x^2}}$

(B) $\frac{-1}{\sqrt{1+x^2}}$

(C) $\frac{1}{\sqrt{1-x^2}}$

(D) $\frac{-1}{\sqrt{1-x^2}}$

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9) If \underline{a} and \underline{b} are parallel vectors then $\underline{a} \times \underline{b} =$

- (A) 1 (B) 0 (C) -1 (D) 2

10) If any two vectors of scalar triple product are equal, then its value is

- (A) 0 (B) 1 (C) 2 (D) -1

11) $\int \frac{\sin 2x}{4 \sin x} dx =$

- (A) $\sin 2x + c$ (B) $2 \sin 2x + c$ (C) $\frac{1}{2} \sin x + c$ (D) $2 \sin x + c$

12) $\int \frac{-1}{x\sqrt{x^2-1}} dx =$

- (A) $\tan^{-1} x + c$ (B) $\operatorname{cosec}^{-1} x + c$ (C) $\sec^{-1} x + c$ (D) $\sin^{-1} x + c$

13) Slope of line perpendicular to $3x - 4y + k = 0$ is

- (A) -1 (B) $\frac{4}{3}$ (C) $\frac{3}{4}$ (D) $-\frac{4}{3}$

14) Distance of line $5x + 12y + 39 = 0$ from $(0, 0)$ is

- (A) 3 (B) 5 (C) 13 (D) 39

15) Point $\left(\frac{3}{7}, \frac{-5}{7}\right)$ lies in quadrant

- (A) I (B) II (C) III (D) IV

16) The point $(1, 2)$ satisfies the inequality

- (A) $x + 2y > 3$ (B) $x - 2y > 3$ (C) $3x + 2y < 3$ (D) $x + 2y < 3$

17) What is the eccentricity of a point circle $x^2 + y^2 = 0$

- (A) $\frac{1}{\sqrt{2}}$ (B) 1 (C) $\sqrt{2}$ (D) 0

18) Length of Latus rectum of a parabola $8x^2 = -32y$ is

- (A) 16 (B) 4 (C) -4 (D) 8

19) The end points of the minor axis of the ellipse are called

- (A) Foci (B) Vertices (C) Co-vertices (D) Directrices

20) A conic is said to be a hyperbola if

- (A) $e = 0$ (B) $e = 1$ (C) $e < 1$ (D) $e > 1$

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Section ----- I

2. Answer briefly any Eight parts from the followings:- 8 × 2 = 16

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- (i) Evaluate $\lim_{x \rightarrow 3} \frac{x-3}{\sqrt{x}-\sqrt{3}}$ (ii) Given $f(x) = x^3 - 2x^2 + 4x - 1$ then find $f(1)$ and $f(1+x)$
- (iii) If $f(x) = 2x^2 + x - 5$ then determine Left hand Limit and Right hand Limit at $x = 1$
- (iv) Differentiate $\frac{2x-3}{2x+1}$ w.r.t x . (v) If $x = 1 - t^2$ and $y = 3t^2 - 2t^3$ then find $\frac{dy}{dx}$
- (vi) Find $\frac{dy}{dx}$ if $4x^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$
- (vii) If $f(x) = \frac{e^{ax} - e^{-ax}}{e^{ax} + e^{-ax}}$ then find $f'(x)$ (viii) Find first four derivative of $\cos(ax + b)$
- (ix) Expand a^x in the Maclaurin's series.
- (x) Find the extreme values of the function $f(x) = 3x^2 - 4x + 5$
- (xi) Indicate solution region by shading the inequality $3x + 7y \geq 21$, $x - y \leq 2$
- (xii) Define problem constraints.

3. Answer briefly any Eight parts from the followings:- 8 × 2 = 16

- (i) Find δy and dy of $y = x^2 + 2x$ when x changes from 2 to 1.8
- (ii) Evaluate indefinite integral $\int \frac{(\sqrt{\theta}-1)^2}{\sqrt{\theta}} d\theta$ (iii) Find $\int \sin^2 x dx$
- (iv) Evaluate $\int \frac{dx}{x(\ln 2x)^3}$ (v) Find $\int \frac{\sin \theta}{1 + \cos^2 \theta} d\theta$
- (vi) Evaluate $\int \frac{\sec^2 x}{\sqrt{\tan x}} dx$ (vii) Find Integral by parts $\int x \ln x dx$
- (viii) Find \vec{OA} where $\vec{AB} = [4, -2]$ and $B(-2, 5)$
- (ix) Write the direction cosine of $\underline{y} = 3\hat{i} - \hat{j} + 2\hat{k}$
- (x) Find $\sin \theta$ if $|\underline{a} \times \underline{b}| = \sqrt{185}$, $|\underline{a}| = \sqrt{26}$, $|\underline{b}| = 3$
- (xi) Calculate the projection of $\underline{a} = \hat{i} - \hat{k}$ along $\underline{b} = \hat{j} + \hat{k}$
- (xii) A force $\underline{F} = 7\hat{i} + 4\hat{j} - 3\hat{k}$ is applied at $P(1, -2, 3)$. Find its moment about point $Q(2, 1, 1)$

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SGD-12-2-23 -- (2) --

9 × 2 = 18

4. Answer briefly any Nine parts from the followings:-

- (i) The points A(-5, 2) and (5, -4) are ends of a diameter of a circle. Find its centre and radius.
- (ii) Show that A(-3, 6), B(3, 2), C(6, 0) are collinear points.
- (iii) Find the equation of a line of it is perpendicular to line with slope -6 and its y-intercept is $\frac{4}{3}$
- (iv) Find the distance between parallel lines $2x - 5y + 13 = 0$, $2x - 5y + 6 = 0$
- (v) Find k so that the line joining A(7, 3) B(k, -6) and the line joining C(-4, 5) and D(-6, 4) are perpendicular.
- (vi) Find the equation of a vertical line through (-5, 3)
- (vii) Find the lines represented by $2x^2 + 3xy - 5y^2 = 0$
- (viii) Find the centre and radius of a circle $5x^2 + 5y^2 + 14x + 12y - 10 = 0$
- (ix) Find the length of Tangent drawn from P(-5, 10) to the circle $5x^2 + 5y^2 + 14x + 12y - 10 = 0$
- (x) Find vertex and directrix of parabola $(x-1)^2 = 8(y+2)$
- (xi) Find the equation of parabola with Focus (2, 5) and directrix is $y = 1$
- (xii) Find Foci and vertices of ellipse $25x^2 + 9y^2 = 225$
- (xiii) Find the equation of hyperbola centre (0, 0), Focus (6, 0), vertex (4, 0)

Section ----- II

(10 × 3 = 30)

Note: Attempt any three questions.

5-(a) Evaluate $\lim_{\theta \rightarrow 0} \frac{\tan \theta - \sin \theta}{\sin^3 \theta}$

(b) Prove that $y \frac{dy}{dx} + x = 0$ if $x = \frac{1-t^2}{1+t^2}$; $y = \frac{2t}{1+t^2}$

6-(a) Find a joint equation of the lines through the origin and perpendicular to the lines $x^2 - 2xy \tan \alpha - y^2 = 0$

(b) Evaluate $\int \operatorname{cosec}^3 x \, dx$

7-(a) Evaluate $\int_0^{\pi/4} \frac{\sec \theta}{\sin \theta + \cos \theta} \, d\theta$

(b) Maximize $f(x) = 2x + 5y$ subject to the constraints $2y - x \leq 8$, $x - y \leq 4$, $x \geq 0$, $y \geq 0$

8-(a) If $x = a(\theta - \sin \theta)$, $y = a(1 + \cos \theta)$, then Show that $y^2 \frac{d^2 y}{dx^2} + a = 0$

(b) Write an equation of the circle that passes through the points A(4, 5), B(-4, -3), C(8, -3)

9-(a) Find equation of ellipse with centre (0, 0), symmetric with both the axes and passing through points (2, 3) and (6, 1)

(b) Prove that in any triangle $c^2 = a^2 + b^2 - 2ab \cos C$

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